Module code: M337 **Complex analysis**

Errata Document

Contents

SUP085755

Administration

Book A

Page 60, Figure 4.13(b)

The hollow dot at the origin should not be there.

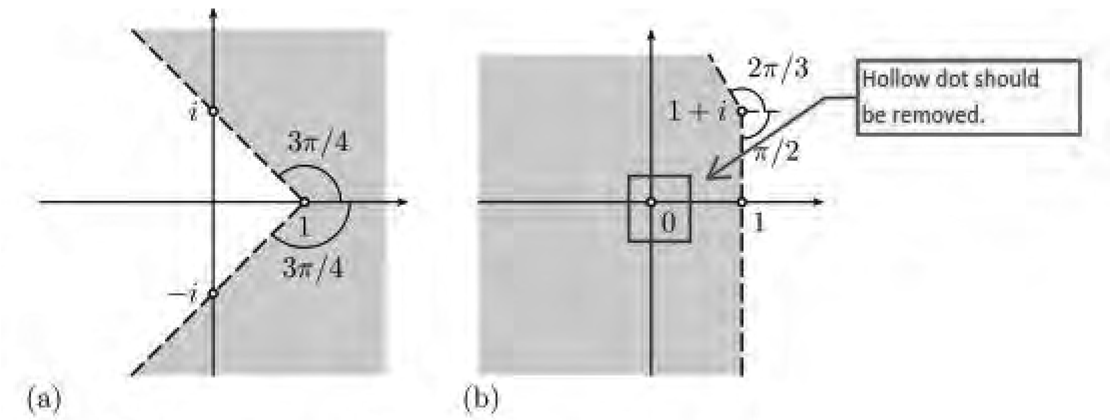

Figure 4.13 Open sectors: (a) $\{z : |\mathrm{Arg}(z-1)| < 3\pi/4\}$, (b) $\{z : \mathrm{Arg}(z-1-i) < -\pi/2 \text{ or } \mathrm{Arg}(z-1-i) > 2\pi/3\}$

Page 289, first displayed equation

This equation says

$$\exp(x+iy) = e^x(\cos x + i\sin y)$$

but it should say

$$\exp(x+iy) = e^x(\cos y + i\sin y).$$

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Book B

Page 19, Theorem 2.1

The theorem is missing the hypotheses that γ_1 and γ_2 have the same initial point and final point, and they are one-to-one on $[a_1, b_1)$ and $[a_2, b_2)$, respectively. The statement has been corrected in the Handbook – see HB B1 2.4, p41.

Theorem 2.1

Let $\gamma_1 : [a_1, b_1] \longrightarrow \mathbb{C}$ and $\gamma_2 : [a_2, b_2] \longrightarrow \mathbb{C}$ be two smooth parametrisations of paths with the same ~~image set Γ, and let~~ f be a function that is continuous on Γ. Then

initial point, final point and image set Γ such that γ_1 and γ_2 are are one-to-one on $[a_1,b_1)$ and $[a_2,b_2)$, respectively. Let

$$\int_\Gamma f(z)\,dz$$

does not depend on which parametrisation γ_1 or γ_2 is used.

Page 120, Solution to Exercise 2.11(b), third line from the end.

The integrand should be $\dfrac{\sin 2z}{z^2+1}$ instead of $\dfrac{\sin 2z}{z^2+i}$.

Formula applied to the two integrals on the right-hand side of equation (S3),

$$\begin{aligned}\int_\Gamma \frac{\sin 2z}{z^2+\underset{1}{\boxed{\cancel{i}}}}\,dz &= \frac{i}{2}\times 2\pi i f(-i) - \frac{i}{2}\times 2\pi i f(i)\\ &= -\pi\sin(-2i)+\pi\sin 2i\\ &= 2\pi i \sinh 2.\end{aligned}$$

(c) Note that $z(z^2-9) = z(z-3)(z+3)$, and that the points 0 and 3 lie inside Γ, but the

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Page 170, fourth and fifth lines of Subsection 3.1

This should say that $F(z) = \operatorname{Log}(1+z)$ is analytic on $\mathbb{C} - \{x \in \mathbb{R} : x \leq -1\}$, not that its domain is $\mathbb{C} - \{x \in \mathbb{R} : x \leq -1\}$. In fact, its domain is $\mathbb{C} - \{-1\}$, however, it is not analytic at any point on the set $\{x \in \mathbb{R} : x \leq -1\}$.

3.1 Taylor series

In Example 2.4 at the end of the previous section you saw that

$$\operatorname{Log}(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \cdots, \quad \text{for } |z| < 1.$$

We say that the function

$$F(z) = \operatorname{Log}(1+z),$$

is analytic on the region

which ~~has domain~~ $\mathbb{C} - \{x \in \mathbb{R} : x \leq -1\}$, is *represented* on the open disc $D = \{z : |z| < 1\}$ by the power series

$$z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \cdots$$

Page 192, Substitution Rule for Power Series

In the first sentence, interchange the two phrases 'powers of z' and 'powers of w'. For clarity, it then makes sense in the second sentence to interchange the two phrases 'powers of z-α' and 'powers of 'powers of w-β', although this is not essential. The statement has been corrected in the Handbook – see HB B3 4.3, p53.

Substitution Rule for Power Series

The substitution

$$w = \lambda z^k, \quad \text{where } \lambda \neq 0,\ k \in \mathbb{N},$$

changes a power series in powers of ~~z~~ w with radius of convergence R to a power series in powers of ~~w~~ z with radius of convergence $\sqrt[k]{R/|\lambda|}$.

The substitution

$$w = z + \beta - \alpha$$

changes a power series in powers of ~~$z - \alpha$~~ $w - \beta$ to a power series in powers of ~~$w - \beta$~~, $z - \alpha$ and preserves the radius of convergence.

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Administration

Book C

The heading of the most recent errata is highlighted red.

Page 37, Example 3.5, sixth line of solution

The displayed equation involves three integrals. The lower limit of the third integral should be $2+\varepsilon$ rather than $2-\varepsilon$.

real axis). So, using the definitions of improper integrals from the previous subsection, we see that

$$\int_{-\infty}^{\infty} f(t)\,dt = \lim_{r\to\infty}\left(\lim_{\varepsilon\to 0}\left(\int_{-r}^{2-\varepsilon} f(t)\,dt + \int_{2+\varepsilon}^{r} f(t)\,dt\right)\right),$$

provided that these limits exist.

1. Let us now think of f as a complex function, replacing the variable t by z. Then f is analytic on the simply connected

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Page 133, Definition box

Replace the phrase 'there is a region $\mathcal{S}$ inside $\mathcal{R}$ with $\alpha \in \mathcal{S}$' with 'inside any open disc in $\mathcal{R}$ centred at α there is a region $\mathcal{S}$ containing α'.

Definition

Let f be a function that is analytic on a region $\mathcal{R}$, and let $\alpha \in \mathcal{R}$. Then f is **n-to-one near α** if inside any open disc in $\mathcal{R}$ centred at α there is a region S containing α ~~there is a region $\mathcal{S}$ inside $\mathcal{R}$ with $\alpha \in \mathcal{S}$~~ such that for each point w in $f(\mathcal{S}) - \{f(\alpha)\}$ there are exactly n points z in $\mathcal{S} - \{\alpha\}$ that satisfy $f(z) = w$.

Page 157, Figure 5.1

The label ζ_4 should be ζ_{10}.

Page 283, Solution to Exercise 2.9, fourth line

The displayed equation involves three expressions separated by two equals symbols. In the first expression the terms $(2i+2)$ and $(z+2)$ should be interchanged. In the second expression the terms $(\infty-1)$ and $(w-1)$ should be interchanged.

Solution to Exercise 2.9

We find the required transformation by using the Implicit Formula for Möbius Transformations, which in this case is interchange both pairs of terms

$$\frac{(z-2)}{\boxed{(2i+2)}}\frac{\boxed{(z+2)}}{(2i-2)} = \frac{(w-i)}{\boxed{(\infty-1)}}\frac{\boxed{(w-1)}}{(\infty-i)} = \frac{w-i}{w-1}.$$

By evaluating the constant term and cross-multiplying, we obtain

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Page 287, Solution to Exercise 3.12(b)(ii), fourth line

The expression $\frac{1}{2}i+i$ should be $\frac{1}{2}+i$.

(ii) An Apollonian form of the equation for the circle C_2 is

$$\left|z-\left(\tfrac{1}{4}+i\right)\right| = k|z-(1+i)|,$$

for some $k>0$. Since $\frac{1}{2}\boxed{i}+i$ lies on C_2,

$$k=\frac{\left|\left(\frac{1}{2}+i\right)-\left(\frac{1}{4}+i\right)\right|}{\left|\left(\frac{1}{2}+i\right)-(1+i)\right|}=\frac{\left|\frac{1}{4}\right|}{\left|-\frac{1}{2}\right|}=\frac{1}{2},$$

Last modified: Sunday, 7 Apr 2019, 18:55

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Book D

Page 61, Subsection 5.1, fourth line

The expression $J_a(c)$ should be $J_a(C)$.

5.1 Aerofoils

In Subsection 3.2 we saw that if $a, b > 0$, then t
a one-to-one mapping of the circle $C = \{w : |w$
aerofoil shaped curve with a cusp at $z = 2a$. Th
boundary $\cancel{J_a(c)}$ $\boxed{J_a(C)}$ is an example of a *Joukowski ae*

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Handbook

The heading of the most recent errata is highlighted red.

Page 68, item 3.3, second line

Replace the phrase 'there is a region $\mathcal{S}$ inside $\mathcal{R}$ with $\alpha \in \mathcal{S}$' with 'inside any open disc in $\mathcal{R}$ centred at α there is a region $\mathcal{S}$ containing α'.

3. Let f be a function that is analytic on a region $\mathcal{R}$, and let $\alpha \in \mathcal{R}$.
inside any open disc in $\mathcal{R}$ centred at α there is a region $\mathcal{S}$ containing
Then f is ***n*-to-one near α** if ~~there is a region $\mathcal{S}$ inside $\mathcal{R}$ with $\alpha \in \mathcal{S}$~~ such that for each point w in $f(\mathcal{S}) - \{f(\alpha)\}$ there are exactly n points z in $\mathcal{S} - \{\alpha\}$ that satisfy $f(z) = w$.

Last modified: Sunday, 7 Apr 2019, 19:00

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